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The dual module of Gorenstein k-algebras

38th Symposium on Commutative Algebra in Japan
9th Japan-Vietnam Joint Seminar on Commutative Algebra

MAIN GOAL

We will survey some recent results on:

an extension of Macaulay's Inverse System theorem to Gorenstein d-dimensional k-algebras

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Consider

$$R = S/I$$

The codimension of R (of I) is defined by

$$codim(R) = dimS - dimR = n - dimR$$

R is said a complete intersection (c.i.) if I can be generated by $codim(R)$ elements.

$$c.i. \implies Gorenstein$$

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Based on the famous paper by H. Bass ('63) (On the ubiquity of Gorenstein rings), there are many equivalent definitions of Gorenstein rings:

Definition. R is Gorenstein if R is Cohen-Macaulay and its dualizing module (or canonical module) $Ext_S^{n-d}(R,S)$ is free (of rank 1) where $d = dimR$.

In terms of free resolutions

Proposition. Let $0 \to F_c \to F_{c-1} \to \cdots \to F_0 \to R \to 0$ a *minimal* free S-resolution of R. Then

$$R \text{ is Gorenstein } \iff c = codim(R) \text{ and } F_c \simeq S$$

$Codim(R) = 2 \qquad 0 \to S \to S^2 \to S \to R \to 0$

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GORENSTEIN RINGS
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Theorem. [Buchsbaum-Eisenbud] $codim(R) = 3$

R is Gorenstein $\iff$ I is generated by $2m$-order Pfaffians of a skew-symmetric $(2m+1)$ alternating matrix A.

In this case a minimal free resolution of R over S has the form

$$0 \to S \to S^{2m+1} \underset{A}{\to} S^{2m+1} \to S \to R \to 0$$

A. Kustin, M. Reid studied the projective resolution of Gorenstein ideals of codimension 4, aiming to extend the previous famous theorem by Buchsbaum and Eisenbud.

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Let $k = \overline{k}$ of arbitrary characteristic.

Let $S = k[[x_1, \ldots, x_n]]$ (or $k[x_1, \ldots, x_n]$) and let $E_S(k)$ the injective hull of k as R-module. Gabriel (58) observed that an injective hull of $k = S/(x_1, \ldots, x_n)$

$$E_S(k) \simeq D^k_{\cdot}(S_1) \simeq k[X_1, \ldots, X_n] := D$$

a divided power ring.

D is a S-module by a contraction action:

$$x_i \circ X^{[a]} = x_i \circ (X_1^{a_1} \cdots X_n^{a_n}) = X_1^{a_1} \cdots X_i^{a_i - 1} \cdots X_n^{a_n}$$

if $a_i > 0$. If $a_i = 0$, then is 0.

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INVERSE SYSTEM

Example:

$$x_1 \circ X_1^2 X_2 = X_1 X_2$$
$$x_1 \circ X_2^2 = 0$$

If we assume $char(k) = 0$, then

$$\begin{array}{ccc} (D, \circ) & \simeq & (k[X_1, \ldots, X_n], \partial) \\ X^{[a]} & \dashrightarrow & \frac{X^{[a]}}{a!} \end{array}$$

where $a! = \prod(a_i!)$ and ∂ is the usual partial derivative (with coefficients).

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We denote

$$\cdot^{\vee} = Hom_S(\cdot, D)$$

the exact functor in the category of the S-modules. Matlis ('58) showed that the functor $\vee$ defines an equivalence between

$$\{\text{Artinian } S\text{-modules}\} \longrightarrow \{\text{Noetherian } S\text{-modules}\}$$

$$S/I \quad \dashrightarrow \quad (S/I)^{\vee} := I^{\perp} = < \{g(X) \in D \mid I \circ g(X) = 0\} >$$

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Example : Let $I = (x^2, y^3) \subset S = k[[x,y]]$. Then $I^\perp$ is a S-submodule of $D = k[X,Y]$ and

$$I^\perp = < \{g \in D \mid x^2 \circ g = 0 \text{ and } y^3 \circ g = 0\} > = < XY^2 >$$

If $I \subset S$ is an ideal (not necessarily 0-dimensional), then

$$(S/I)^\vee = Hom_S(R/I, D) \simeq I^\perp = < \{g(X) \in D \mid I \circ g(X) = 0\} >,$$

a S-submodule of D and called Macaulay's inverse system of I.

$$I^\perp \text{ is finitely generated} \iff S/I \text{ is 0-dimensional}$$

$$S/I \text{ is 0-dimensional Gorenstein} \iff I^\perp \text{ is cyclic.}$$

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Macaulay proved that there is the following 1-1 correspondence

$$\left\{ \begin{array}{c} I \subseteq S \text{ ideal such that} \\ S/I \text{ is Artinian Gorenstein} \\ \text{with socledegree}(S/I) = s. \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} M = S \circ F \\ S\text{-cyclic submodule of } D \\ \text{with degree } F = s \end{array} \right\}$$

$$\begin{array}{ccc} I & \longrightarrow & I^{\perp} \\ Ann_S(F) & \longleftarrow & M = \langle F \rangle_S \end{array}$$

Given a S-submodule M of D then

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Example 1. Let $F = X^2 + Y^3 \in D = k[X, Y]$ and let $S = k[[x, y]]$. Then

$$I = Ann_S(F) = (xy, x^2 - y^3)$$

and $R = S/I$ is Gorenstein c.i.

$$e = \ell(S/I) = \dim_k < F > = \dim_k < F, Y^2, X, Y, 1 >= 5.$$

$$HF_{S/I}(j) = \dim_k(I^{\perp})_j \ : \ h = (1, 2, 1, 1)$$

Example 2. Let $F = X^2 + Y^2 + Z^2 \in D = k[X, Y, Z]$ and let $S = k[[x, y, z]]$. Then

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EXAMPLES

Macaulay Inverse System

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This topic plays an important role in studying the Hilbert scheme $Hilb_d(\mathbb{P}^n)$ parameterizing Gorenstein 0-dimensional subschemes of $\mathbb{P}^n$ and the rationality of the Poincaré series of k as R-module, see [Emsalem], [Iarrobino], [Poonen], [Erman], [Cartwright-Erman-Velasco-Viray], [-, Sega]

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Proposition. Let $A = S/I$ and $B = S/J$ be two local Artinian Gorenstein algebras so that $I = Ann_S(F)$ and $J = Ann_S(G)$ with $F, G \in D$. Then TFAE:

- 1. $A \simeq B$
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Notation:
$L = (l_1, \ldots, l_d) \in \mathbb{N}^d_+$
$\gamma_i = (0, \ldots, 0, \underset{i}{1}, 0, \ldots, 0)$
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Definition. Let $d > 0$ and let $M \neq (0)$ be a S-submodule of the $D = E_S(k)$. We say that M is G_d-admissible, $1 \leq d < n$, if it admits a system of generators $\{H_L\}_{L \in \mathbb{N}^d_+}$ in $D = k[Z_1, \ldots, Z_n]$ satisfying for every $L \in \mathbb{N}^d_+$ and $i = 1, \ldots, d$ the following conditions:

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There is a one-to-one correspondence between the following sets:

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d-dimensional Gorenstein rings

Main Theorem (J. Elias, R.)

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We present two 1-dimensional examples starting from the same Artinian reduction $S/Ann_S(H_1)$ where $S = k[[x, y, z]]$ and

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EXAMPLES

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We present two 1-dimensional examples starting from the same Artinian reduction $S/Ann_S(H_1)$ where $S = k[[x, y, z]]$ and

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Theorem [Elias,—]

If $M = \langle H_L, L \in \mathbb{N}^d_+ \rangle \subseteq D$ *is a homogeneous* G_d*-admissible* S*-submodule of* D*, then*

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EXAMPLE

ELLIPTIC CURVE IN $\mathbf{P}^4_k$.

Let $H_{11} = X^2 + Y^2 + XZ \in D = k[X, Y, Z, T, W]$ $(codim = 3,\ d = 2)$

Notice that $e = \dim_k < H_{11} >= \dim_k < H_1, X + Z, Y, X, 1 >= 5$ and $r = \deg H_{11} = 2$.

We may construct

$$H_{22} = TW\ H_{11} + C_2,$$

$$H_{33} = TW\ H_{22} + C_3,$$

$$H_{44} = TW\ H_{33} + C_4$$

with $C_i \in k[T, W]$ satisfying the conditions and by the previous result

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$R = S/I$ is a two-dimensional Gorenstein ring of multiplicity 5, $\{w, t\}$ is a regular sequence in S/I.

The projective scheme C defined by S/I is a non-singular arithmetically Gorenstein elliptic curve of $\mathbb{P}^4_k$.

The generators of I are the Pfaffians of the skew matrix

$$\begin{pmatrix} 0 & -x+t & -t & x & -y \\ x-t & 0 & x & -y & z+t \\ t & -x & 0 & z+w & 0 \\ -x & y & -z-w & 0 & -t \\ y & -z-t & 0 & t & 0 \end{pmatrix}$$

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THANK YOU FOR THE ATTENTION!